Physics 409 **Problem #4—Solving Poisson's Equation via Overrelaxation** Fall 2002

Poisson's equation is a partial differential equation relating changes in the electrostatic potential to an arbitrary charge distribution. For this assignment, you will use a spreadsheet to perform a numerical solution of the two-dimensional Poisson equation using the iterative overrelaxation method. As explained in class, the solution is obtained by iteratively calculating the value of each grid cell based on its specified charge and the current values of the cell and its immediate neighbors. The standard relaxation method uses

$$\varphi_{ij}^{r} = \left(Q_{ij} + \varphi_{i-1,j} + \varphi_{i+1,j} + \varphi_{i,j-1} + \varphi_{i,j+1}\right)/4$$

while the overrelaxation method uses

$$\varphi_{ij}^{or} = (1-r)\varphi_{ij} + r\varphi_{ij}^{r}$$

where r is an "overrelaxation parameter" that is set to a value large enough to produce rapid convergence without causing instability in the iterative process.

Once φ has converged everywhere, we can use Poisson's equation directly to find (or check!) the charge distribution using

$$Q_{ij} = 4\varphi_{ij} - \varphi_{i-1,j} - \varphi_{i+1,j} - \varphi_{i,j-1} - \varphi_{i,j+1}$$

Your assignment is to use a spreadsheet to solve for the potential in a 2-d rectangular region subject to arbitrarily specified boundary values (either fixed or "floating" as described in class) and an arbitrarily specified charge distribution. To do this

1. Set up a pair of rectangular "input grids" (at *least* 20x20) for the boundary values of the potential and any specified charge distribution.
2. Set up a "solution grid" for the potential which implements the overrelaxation method referring to the input grids for values of φ on the boundaries and any interior points and any nonzero values of Q.
3. Set up a "derived charge" grid that determines the values of Q at each grid point.
4. Set up three-d surface plots for $\varphi(x,y)$ and $Q(x,y)$.

Once your basic spreadsheet is complete, save it with all boundary values of φ and all values of Q set equal to zero so that it can be easily opened and adjusted for any other specified values.

Now comes the fun part! Use your spreadsheet to

1. Find the potential and charge distribution resulting from a conducting hollow square box near the center of your grid charged to some constant potential (say "10") with the boundary values for the region set to 0. Note that the boundaries *should* be as far as possible from the box if we want to properly simulate the potential due to an *isolated* conductor. Comment on both the potential inside the box and the observed charge distribution. Why are they the way they are?
2. Do the same for a uniformly charged "cylinder." Note that, in this case, you will specify some constant value in a roughly circular region of the charge input grid. Also in this case the "derived charge" grid should serve primarily as a check on the solution, reproducing the input distribution. Comment on the observed potential *inside* the "cylinder."
3. Find the potential for a pair of identical charges or conductors by letting the right boundary sit along the perpendicular bisector of the two objects and setting its boundary to the "floating" condition. Explain why the floating condition is appropriate and comment on the implications of your result for the electric field in the region.
4. Do the same for *four* identical charges by setting *both* the right and lower boundaries to the "floating" condition.
5. Find the potential and charge distribution resulting from a parallel plate capacitor in two ways: First, by setting up the two plates in the solution region. Second, by setting up one plate near a boundary and setting the boundary conditions appropriately to reflect the presence of an equal but oppositely charged conductor on the other side of the boundary. Comment on the observed charge distribution and on the implied electric field.
6. Try to create an arbitrary region that has a constant potential by adjusting *only* the input charge distribution in that region. Comment on how you had to adjust the charge distribution to do so and what that means physically.
7. Find at *least* one other interesting set of input specifications and comment on what it represents and why the results make sense.

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