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A Large Mass Hierarchy from a Small Extra Dimension

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Abstract

We propose a new higher-dimensional mechanism for solving the hierarchy problem. The weak scale is generated from a large scale of order the Planck scale through an exponential hierarchy. However, this exponential arises not from gauge interactions but from the background metric (which is a slice of AdS_5 spacetime). This mechanism relies on the existence of only a single additional dimension. We demonstrate a simple explicit example of this mechanism with two three-branes, one of which contains the Standard Model fields. The experimental consequences of this scenario are new and dramatic. There are fundamental spin-2 excitations with mass of weak scale order, which are coupled with *weak scale* as opposed to gravitational strength to the standard model particles. The phenomenology of these models is quite distinct from that of large extra dimension scenarios; none of the current constraints on theories with very large extra dimensions apply.

1 Introduction

If spacetime is fundamentally higher dimensional with $4+n$ spacetime dimensions, then the effective four-dimensional (reduced) Planck scale, $M_{Pl} = 2 \times 10^{18}$ GeV, is determined by the fundamental $(4+n)$-dimensional Planck scale, M, and the geometry of the extra dimensions. In the simplest cases, where the higher dimensional spacetime is approximately a product of a 4-dimensional spacetime with a n-dimensional compact space,

$$M_{Pl}^2 = M^{n+2} V_n, \tag{1}$$

where V_n is the volume of the compact space. Recently it has been proposed that the large hierarchy between the weak scale and the fundamental scale of gravity can be eliminated by taking the compact space to be very large [1]. The fact that we do not see experimental signs of the extra dimensions despite the fact that the compactification scale, $\mu_c \sim 1/V_n^{1/n}$, would have to be much smaller than the weak scale, implies that the SM particles and forces with the exception of gravity are confined to a 4-dimensional subspace within the $(4+n)$-dimensional spacetime, referred to as a "3-brane". While this scenario does eliminate the hierarchy between the weak scale v and the Planck scale M_{Pl}, it introduces a new hierarchy, namely that between μ_c and v. In light of this it is worthwhile to explore alternatives.

Here we will present a distinct higher dimensional scenario which provides an alternative approach to generating the hierarchy. We propose that the metric is not factorizable, but rather the four-dimensional metric is multiplied by a "warp" factor which is a rapidly changing function of an additional dimension. The dramatic consequences for the hierarchy problem that we identify in this letter follow from the particular non-factorizable metric,

$$ds^2 = e^{-2kr_c\phi}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2, \tag{2}$$

where k is a scale of order the Planck scale, x^μ are coordinates for the familiar four dimensions, while $0 \leq \phi \leq \pi$ is the coordinate for an extra dimension, which is a finite interval whose size is set by r_c. We will show that this metric is a solution to Einstein's equations in a simple set-up with two 3-branes and appropriate cosmological terms. In this space, four-dimensional mass scales are related to five-dimensional input mass parameters and the warp factor, $e^{-2kr_c\phi}$. To generate a large hierarchy does not require extremely large r_c. This is because the source of the hierarchy is an *exponential* function of the compactification radius. The small exponential factor above is the source of the large hierarchy between the observed Planck and weak scales.

Although designed to address the hierarchy problem by exploiting an additional dimension, this solution is quite distinct from that proposed in Ref. [1]: 1) The hierarchy between the fundamental five-dimensional Planck scale and the compactification scale, $\mu_c \equiv 1/r_c$ is only of order 50, as opposed to $(M_{Pl}/TeV)^{2/n}$. 2) There is one additional dimension, as opposed to $n \geq 2$. The experimentally distinctive consequences are: 1) There are no light Kaluza-Klein modes. The excitation scale is of order a TeV. Therefore, current constraints from particle physics [2], astrophysics and cosmology [3] do not apply. Because of this the scale at which gravity becomes strong can be quite low. However, as with the scenario of

Ref. [1], string/M-theoretic excitations are also expected to appear at the TeV scale. 2) The coupling of an individual KK excitation to matter or to other gravitational modes is set by the weak, not the Planck scale. The KK modes are *not* invisible; they should be observable at high energy colliders as spin-2 resonances that can be reconstructed from their decay products.

2 The Set-Up

Because our spacetime does not fill out all of five dimensions, we need to specify boundary conditions, which we take to be periodicity in ϕ, the angular coordinate parameterizing the fifth dimension, supplemented with the identification of (x,ϕ) with $(x,-\phi)$; that is we work on the space $S^1/\mathbf{Z_2}$. We take the range of ϕ to be from $-\pi$ to π; however the metic is completely specified by the values in the range $0 \leq \phi \leq \pi$. The orbifold fixed points at $\phi = 0, \pi$ will be taken as the locations of two 3-branes, extending in the x^μ-directions, so that they are the boundaries of the five-dimensional spacetime. The 3-branes can support $(3+1)$-dimensional field theories. Both couple to the purely four-dimensional components of the bulk metric:

$$g^{vis}_{\mu\nu}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = \pi), \quad g^{hid}_{\mu\nu}(x^\mu) \equiv G_{\mu\nu}(x^\mu, \phi = 0), \tag{3}$$

where G_{MN}, $M, N = \mu, \phi$, is the five-dimensional metric.

This set-up is in fact similar to the scenario of Ref. [1]. However, we take into account the effect of the branes on the bulk gravitational metric. Working out the consequences of the localized energy density peculiar to the brane set-up, we find a new solution to the hierarchy problem. As we will show, this requires nothing beyond the existence of the 3-branes in five dimensions and their compatibility with four-dimensional Poincare invariance.

The classical action describing the above set-up is given by

$$\begin{aligned} S &= S_{gravity} + S_{vis} + S_{hid} \\ S_{gravity} &= \int d^4x \int_{-\pi}^{\pi} d\phi \sqrt{-G}\{-\Lambda + 2M^3 R\} \\ S_{vis} &= \int d^4x \sqrt{-g_{vis}}\{\mathcal{L}_{vis} - V_{vis}\} \\ S_{hid} &= \int d^4x \sqrt{-g_{hid}}\{\mathcal{L}_{hid} - V_{hid}\}. \end{aligned} \tag{4}$$

Note that from each 3-brane Lagrangian we have separated out a constant "vacuum energy" which acts as a gravitational source even in the absence of particle excitations. The detailed form of the rest of the 3-brane Lagrangians will not be relevant for determining the classical five-dimensional metric in the ground state. Further discussion of 3-brane actions can be found in Ref. [6].

3 Classical Solution

In this section we solve the five-dimensional Einstein's equations for the above action,

$$\sqrt{-G}\left(R_{MN}-\frac{1}{2}G_{MN}R\right) = -\frac{1}{4M^3}[\Lambda\sqrt{-G}\ G_{MN}+V_{vis}\sqrt{-g_{vis}}\ g^{vis}_{\mu\nu}\ \delta^\mu_M\delta^\nu_N\ \delta(\phi-\pi) + V_{hid}\sqrt{-g_{hid}}\ g^{hid}_{\mu\nu}\ \delta^\mu_M\delta^\nu_N\ \delta(\phi)]. \quad (5)$$

We assume there exists a solution that respects *four*-dimensional Poincare invariance in the x^μ-directions. A five-dimensional metric satisfying this ansatz takes the form

$$ds^2 = e^{-2\sigma(\phi)}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2. \quad (6)$$

The coefficient, r_c, is independent of ϕ when we have chosen coordinates such that ϕ is proportional to the proper distance in the extra direction, r_c being the constant of proportionality. In this way, r_c is the "compactification radius" of the extra dimensional circle prior to orbifolding. After orbifolding, the size of the extra dimensional interval is πr_c. r_c is independent of x by four-dimensional Poincare invariance.

With this ansatz, the Einstein's equations following from Eq. (5) reduce to

$$\frac{6\sigma'^2}{r_c^2} = \frac{-\Lambda}{4M^3}, \quad (7)$$

$$\frac{3\sigma''}{r_c^2} = \frac{V_{hid}}{4M^3 r_c}\delta(\phi) + \frac{V_{vis}}{4M^3 r_c}\delta(\phi-\pi). \quad (8)$$

The solution to Eq. (7) consistent with the orbifold symmetry $\phi \to -\phi$ is

$$\sigma = r_c|\phi|\sqrt{\frac{-\Lambda}{24M^3}}. \quad (9)$$

The additive integration constant has been omitted because it just amounts to an overall constant rescaling of the x^μ. Clearly this solution only makes sense if $\Lambda < 0$, which we will therefore assume to be the case from now on. Note that the spacetime in between the two 3-branes is simply a slice of an AdS_5 geometry.[1]

Recall that in computing derivatives we are to consider the metric a periodic function in ϕ. Eq. (9), valid for $-\pi \le \phi \le \pi$, then implies

$$\sigma'' = 2r_c\sqrt{\frac{-\Lambda}{24M^3}}[\delta(\phi)-\delta(\phi-\pi)]. \quad (10)$$

From this we see that we only obtain a solution to Eq. (8) if $V_{hid}, V_{vis}, \Lambda$ are related in terms of a single scale k,

$$V_{hid} = -V_{vis} = 24M^3k, \quad \Lambda = -24M^3k^2. \quad (11)$$

[1]Note, this makes our bulk gravitational dynamics compatible with a supersymmetric extension.

These relations between the boundary and bulk cosmological terms are required in order to obtain a solution that respects four-dimensional Poincare invariance. Note that precisely these relations arise in the five-dimensional effective theory of the Horava-Witten scenario if one were to interpret the expectation values of the background 3-form field (but with frozen Calabi-Yau moduli) as cosmological terms in the effective five dimensional theory after Calabi-Yau compactification [5]. We will assume that $k < M$ so that the bulk curvature is small compared to the higher dimensional Planck scale and we trust our solution.

Our final solution for the bulk metric is then

$$ds^2 = e^{-2kr_c|\phi|}\eta_{\mu\nu}dx^\mu dx^\nu + r_c^2 d\phi^2. \tag{12}$$

The compactification radius r_c is effectively an arbitrary integration constant for this solution.

4 Physical Implications

We are considering a small r_c (but still larger than $1/k$). Therefore, the fifth dimension cannot be resolved in present (or future) gravity experiments; spacetime appears four-dimensional. It therefore makes sense to use a four-dimensional effective field theory description. In this section, we derive the parameters of this low-energy theory, namely the four-dimensional Planck scale, and the mass parameters of the four-dimensional fields, in terms of the five-dimensional scales, M, k, r_c.

The first step is to identify the massless gravitational fluctuations about our classical solution, Eq. (12). These will provide the gravitational fields for our effective theory. They are the zero-modes of our classical solution, and take the form

$$ds^2 = e^{-2kT(x)|\phi|}[\eta_{\mu\nu} + \overline{h}_{\mu\nu}(x)]dx^\mu dx^\nu + T^2(x)d\phi^2. \tag{13}$$

Here, $\bar{h}_{\mu\nu}$ represents tensor fluctuations about Minkowski space and is the physical graviton of the four-dimensional effective theory (and is the massless mode in the Kaluza-Klein decomposition of $G_{\mu\nu}$). Note that this metric is *locally* the same as our "vacuum" solution, Eq. (12), since any smooth four-dimensional metric,

$$\overline{g}_{\mu\nu}(x) \equiv \eta_{\mu\nu} + \overline{h}_{\mu\nu}(x), \tag{14}$$

is locally Minkowskian, while any smooth real function $T(x)$ is locally constant. The compactification radius, r_c, is the vacuum expectation value of the modulus field, $T(x)$. As with many higher dimensional theories, it will be critical that the T modulus is stabilized at its vacuum expectation value r_c, with a mass of at least 10^{-4} eV. Although an essential element of the theory, this problem is not yet solved (but see Refs. [8]). From now on, we replace T with r_c. In compactifying extra dimensions, one frequently encounters vector zero modes from $A_\mu dx^\mu d\phi$ fluctuations of the metric (that is the original Kaluza-Klein idea), corresponding to the continuous isometries of the higher dimensions, but in the present case there are no such isometries in the presence of the 3-branes. So all such off-diagonal fluctuations of the metric are massive and excluded from the low-energy effective theory.

The four-dimensional effective theory now follows by substituting Eq. (13) into the original action, Eq. (4). We focus on the curvature term from which we can derive the scale of gravitational interactions:

$$S_{eff} \supset \int d^4x \int_{-\pi}^{\pi} d\phi \; 2M^3 r_c e^{-2kr_c|\phi|} \sqrt{-\overline{g}} \; \overline{R} \tag{15}$$

where $\overline{R}$ denotes the four-dimensional Ricci scalar made out of $\overline{g}_{\mu\nu}(x)$, in contrast to the five-dimensional Ricci scalar, R, made out of $G_{MN}(x,\phi)$. Because the low-energy fluctuations do not change the ϕ dependence (the effective fields depend on x alone), we can explicitly perform the ϕ integral to obtain a purely four-dimensional action. From this we derive

$$M_{Pl}^2 = M^3 r_c \int_{-\pi}^{\pi} d\phi e^{-2kr_c|\phi|} = \frac{M^3}{k}[1 - e^{-2kr_c\pi}]. \tag{16}$$

This is an important result. It tells us that M_{Pl} depends only weakly on r_c in the large kr_c limit. Although the exponential has very little effect in determining the Planck scale, we will now see that it plays a crucial role in the determination of the visible sector masses.

In order to determine the matter field Lagrangian we need to know the coupling of the 3-brane fields to the low-energy gravitational fields, in particular the metric, $\overline{g}_{\mu\nu}(x)$. From Eq. (3) we see that $g_{hid} = \overline{g}_{\mu\nu}$. This is not the case for the visible sector fields; by Eq. (3), we have $g^{vis}_{\mu\nu} = e^{-2kr_c\pi}\overline{g}_{\mu\nu}$. By properly normalizing the fields we can determine the physical masses. Consider for example a fundamental Higgs field,

$$S_{vis} \supset \int d^4x \sqrt{-g_{vis}} \{ g^{\mu\nu}_{vis} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2 \}, \tag{17}$$

which contains one mass parameter v_0. Substituting Eq. (3) into this action yields

$$S_{vis} \supset \int d^4x \sqrt{-\overline{g}} e^{-4kr_c\pi} \{ \overline{g}^{\mu\nu} e^{2kr_c\pi} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - v_0^2)^2 \}, \tag{18}$$

After wave-function renormalization, $H \to e^{kr_c\pi} H$, we obtain

$$S_{eff} \supset \int d^4x \sqrt{-\overline{g}} \{ \overline{g}^{\mu\nu} D_\mu H^\dagger D_\nu H - \lambda(|H|^2 - e^{-2kr_c\pi} v_0^2)^2 \}. \tag{19}$$

A remarkable thing has happened. We see that the physical mass scales are set by a symmetry-breaking scale,

$$v \equiv e^{-kr_c\pi} v_0. \tag{20}$$

This result is completely general: any mass parameter m_0 on the visible 3-brane in the fundamental higher-dimensional theory will correspond to a physical mass

$$m \equiv e^{-kr_c\pi} m_0 \tag{21}$$

when measured with the metric $\overline{g}_{\mu\nu}$, which is the metric that appears in the effective Einstein action, since all operators get rescaled according to their four-dimensional conformal weight. If $e^{kr_c\pi}$ is of order 10^{15}, this mechanism produces TeV physical mass scales from fundamental mass parameters not far from the Planck scale, 10^{19} GeV. Because this geometric factor is an exponential, we clearly do not require very large hierarchies among the fundamental parameters, v_0, k, M, and $\mu_c \equiv 1/r_c$; in fact, we only require $kr_c \approx 50$.

Having established the relevant masses for matter fields, we turn to the question of the gravitational modes themselves. This gives rise to a rich and very distinctive phenomenology. To determine the parameters of the gravitational modes in detail, requires an explicit Kaluza-Klein decomposition. We will do this in Ref. [9]. The result is that the masses *and* couplings of the Kaluza-Klein modes are determined by the TeV scale. This result can be readily understood.

Until this point, we have viewed $M \approx M_{Pl}$ as the fundamental scale, and the TeV scale as a derived scale as a consequence of the exponential factor appearing in the metric. However, one could equally well have regarded the TeV scale as fundamental, and the Planck scale of 10^{19} GeV as the derived scale. That is, the ratio is the physical dimensionless quantity. From this viewpoint, which is the one naturally taken by a four-dimensional observer residing on the visible brane, the large Planck scale (the weakness of gravity) arises because of the small overlap of the graviton wave function in the fifth dimension (which is the warp factor) with our brane. In fact, this is the *only* small number produced. All other scales are set by the TeV scale.

Technically, this change in viewpoint is established by the change of coordinates, $x^\mu \to e^{kr_c\pi}x^\mu$. In this case, the warp factor at $\phi = \pi$ is unity, whereas that at $\phi = 0$ is $e^{2kr_c}\pi$. In this language, since there is no rescaling of the "v" parameter in the Higgs potential because the Higgs is already canonically normalized, the scale v should take its physical value. Because we are assuming all fundamental mass parameters are of the same order, all these parameters are also of order TeV [2].

This result contrasts sharply with the scenario of large extra dimensions for solving the hierarchy problem with a product structure for the full spacetime, where the Kaluza-Klein splittings are much smaller than the weak scale, possibly smaller than an eV. The dangerous astrophysical and cosmological effects of very light Kaluza-Klein states are absent in our model.

The phenomenological implications of this scenario for future collider searches are very distinctive. For a product spacetime, each excited state couples with gravitational strength, and the key to observing these states in accelerator experiments is the large multiplicity of states due to their fine splittings. In our model, with roughly weak scale splittings a relatively small number of excitations will be kinematically accessible at accelerators. However their couplings to matter are set by the weak scale rather than the Planck scale. Instead of gravitational strength couplings $\sim$ Energy/M_{Pl}, each excited state coupling is of order Energy/TeV, and therefore each can be *individually* detected. These resonances can be de-

[2]Note that the relation between the mass parameters in the new coordinates and the old mass parameters is due to the spacetime coordinate rescaling.

tected via their decay products. This should allow detailed reconstruction, permitting mass and spin determination of these *gravitational* modes.

From the above discussion it should be clear that at energies somewhat larger than the weak scale, the excited gravitons are strongly coupled. This regime should likely open up the production of string/M-theoretic excitations which lie outside the domain of even our starting five-dimensional field theory. This means that although the fundamental scales of the higher dimensional theory are of order M_{Pl}, the *apparent* scale where the theory becomes strongly coupled and the string/M excitations appear is of order the weak scale according to a four-dimensional observer. This is an important result for the consistency of our scenario beyond tree level. As with Ref. [1], the TeV-scale strings will cut off large renormalization of the weak scale.

5 Conclusions

In the 3-brane scenario, where extra-dimensional translational symmetry is necessarily broken, non-trivial warp factors naturally arise upon solving Einstein's equations. The Kaluza-Klein reduction is considerably more subtle than in product spacetimes, as we will detail in a following paper [9]. This has important phenomenological and theoretical implications.

In this letter, we focussed on a potential phenomenological implication of this scenario, namely an exponential generation of the hierarchy. Remarkably, the four-dimensional masses on the visible brane depend on the background metric in such a way that their physical values differ significantly from the input mass parameters, even without invoking a large compactification volume. This is a potential resolution to the hierarchy problem akin in spirit to the ideas of strongly coupled gauge theories which generate the low scale through an exponential times a fundamental high energy scale. As an aside, we mention that the exponential we exploited could generate other scales, such as the low-energy supersymmetry breaking scale. However, it is important to the viability of our mechanism that it is possible to stabilize the compactification radius roughly two orders of magnitude larger than the fundamental five dimensional Planck length. Issues such as flavor violation and proton decay in the face of the low scale of new physics [10], also remain important challenges.

Fortunately, this solution to the hierarchy problem is subject to experimental verification. The phenomenology is quite distinct from the scenario of large radius compactification. The gravitational resonances are of order a TeV, and couple with TeV suppressed, rather than Planck-suppressed, strength. Furthermore, there are no experimental bounds pushing this scale very high. Should this solution prove correct, there is a rich spectroscopy awaiting us at the LHC.

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